

One-loop effective potential for the vacuum gauge field in $M_3 \times S^3 \times S^1$ space-time

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Abstract

We calculate the effective potential for the vacuum gauge field from the one-loop matter effect in $M_3 \times S^3 \times S^1$ space-time. This background geometry is motivated from the recent studies on gauged supergravities with a positive-definite potential, which admits a generalized Kaluza-Klein reduction. We investigate how symmetry breaking patterns through the Hosotani mechanism are affected by the ratio of the radii of S^1 and S^3 .

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I. INTRODUCTION

Recently there has been much interest in a dynamical gauge symmetry breaking mechanism in higher dimensional theories, *i.e.*, the Hosotani mechanism considered on various compact extra dimensions [1]. In this mechanism a constant vacuum gauge field on a multiply-connected space or an orbifold, which is related with the Wilson line element on the space, becomes dynamical degrees of freedom and develops the vacuum expectation value, which plays a role of an “order parameter” for gauge symmetry breaking. Moreover the analysis of the quantum effects in terms of the one-loop effective potential is crucial for determining the order parameter.

For the non-local quantity like the Wilson line element, the effective potential for the vacuum gauge field does not depend on the ultraviolet effects so that the massive particles seem to affect the gauge symmetry breaking patterns through the Hosotani mechanism. In fact, Takenaga pointed out that the existence of bare mass terms changes the gauge symmetry breaking patterns [2].

In this paper, we consider how the symmetry patterns through Hosotani mechanism are affected by the ratio of the radii of S^1 and S^3 in $M_3 \times S^3 \times S^1$ space-time.

This background geometry is motivated from the recent studies on gauged supergravities with a positive-definite potential [3, 4, 5, 6, 7]. These studies were followed by a generalized Kaluza-Klein reduction [8], leading to a positive-definite potential. Moreover the generalized Kaluza-Klein procedure is studied in arbitrary dimensions in [7]. In this method, the generalized Kaluza-Klein reduction was carried out on the bosonic content of the half-maximal supergravity in $D = d+1 (\leq 10)$ which consists of the graviton, the antisymmetric tensor, the vector field and the dilaton, which appear in the bosonic parts of the heterotic string theory (or the NS-NS sector of the type-II string theory). The various d dimensional supergravities resulted from the generalized Kaluza-Klein reduction have spontaneously compactified solutions to $M_{d-3} \times S^3$ and $M_{d-2} \times S^2$ vacuum solutions, assuming that the antisymmetric tensor field has a monopole configuration on S^3 and S^2 , which guarantees that the rest of the space-time is flat.

In this paper, we consider a toy model for the Hosotani mechanism in the space-time motivated by the seven-dimensional gauged supergravity with a positive-definite potential, admitting a spontaneous compactification to a $M_3 \times S^3 \times S^1$ vacuum solution. We assume

that the antisymmetric tensor field on S^3 forms the magnetic flux configuration, while there is a gauge field on S^1 which becomes dynamical degrees of freedom, causing a symmetry breaking under a certain condition that appeared in the Hosotani mechanism. We further focus on our attention to the possibility of the symmetry breaking without specifying a group representation.

In such a background space-time, we consider quantum effects of a scalar field as well as a fermion without a coupling to the flux on S^3 . In this setting, we calculate the one-loop effective potential for the vacuum gauge field in $M_3 \times S^3 \times S^1$ space-time. In the present model, there are no massless fermion in the low-energy effective theory. Thus the effects of the non-zero eigenmodes of S^3 may be crucial for the symmetry breaking patterns.

The organization of this paper is as follows; in the next section, in order to estimate the effective potential for the vacuum gauge field on S^1 , we compute the one-loop effects of matter fields in $M_3 \times S^3 \times S^1$ space-time. We investigate how the symmetry breaking patterns through the Hosotani mechanism are affected by changing the ratio of radii of S^1 and S^3 , after carried out the regularization and the summation over the massive modes. Finally we discuss the result in the last section.

II. ONE-LOOP EFFECTIVE POTENTIAL

In this section, we consider a toy model motivated by the seven-dimensional gauged supergravity with a positive-definite potential, permitting a vacuum solution whose space-time geometry is $M_3 \times S^3 \times S^1$. We compute the quantum effects of the scalar and fermion field with the vacuum gauge field in this background solution. We restrict our attention to whether the symmetry is broken or not, without concerning about the symmetries realized.

A. Scalar field

Firstly, we compute the one-loop quantum effect of the scalar field in $M_3 \times S^3 \times S^1$ space-time. We assume that the scalar field is decoupled from the flux on S^3 . In general, as is denoted in [9], the effective vacuum energy density V_0 in a background geometry which is $M_{(d+1)} \times S^N$ space-time [($d+1$)-dimensional Minkowski space-time \times N -sphere] is represented

by

$$\begin{aligned} V_0 &\propto \frac{1}{2} \ln \det[p^2 + m^2] \\ &= \frac{1}{2} \int \frac{d^{d+1}p}{(2\pi)^{d+1}} \sum_{l=0}^{\infty} D_l^{\text{sca}}(N) \ln \left(p^2 - \frac{\Lambda_l^{\text{sca}}(N)}{a^2} + m^2 \right), \end{aligned} \quad (1)$$

$$= -\frac{1}{2} \int_0^{\infty} \frac{dt}{(4\pi)^{\frac{d+1}{2}}} \frac{1}{t^{\frac{d+1}{2}+1}} \sum_{l=0}^{\infty} D_l^{\text{sca}}(N) \exp \left[\left(\frac{\Lambda_l^{\text{sca}}(N)}{a^2} - m^2 \right) t \right], \quad (2)$$

for a single scalar degree of freedom. Here a is the radius of S^N and m is a bare mass. $D_l^{\text{sca}}(N)$ and $-\Lambda_l^{\text{sca}}(N)$ denote the degeneracies and the eigenvalues of the laplacian on the unit N -sphere, respectively.

For a scalar field, the eigenvalues of the laplacian and the degeneracies on S^N are obtained by

$$\Lambda_l^{\text{sca}}(n) = -l(l+n+1), \quad D_l^{\text{sca}}(n) = \frac{(2l+n+1)\Gamma(l+n-1)}{l!\Gamma(n)}, \quad (3)$$

respectively.

In addition, if we consider the S^1 compactification, we take the additional discrete eigenvalues n^2/b^2 , where n is integer and b is the radius of S^1 . Then the one-loop vacuum energy in $M_3 \times S^3 \times S^1$ space-time can be written as

$$-\frac{1}{2(4\pi)^{\frac{3}{2}}} \int_0^{\infty} \frac{dt}{t^{\frac{5}{2}}} \sum_{l=0}^{\infty} \sum_{n=-\infty}^{\infty} D_l^{\text{sca}}(3) \exp \left[\left(\frac{\Lambda_l^{\text{sca}}(3)}{a^2} - m^2 \right) t \right] \exp \left[-\frac{n^2}{b^2} t \right]. \quad (4)$$

Now we consider the vacuum gauge field on S^1 . Once we choose a symmetry group and a group representation of the scalar field, the coupling to the vacuum gauge field is determined and the eigenvalues on S^1 is specified for each component field. However, if we concentrate ourselves on the *possibility* of the symmetry breaking, we need not specify the representation. We assume that the spectrum of the scalar component coupled to the non-zero vacuum gauge field should take the following form

$$\frac{(n-v)^2}{b^2}, \quad (5)$$

where v denotes a characteristic scale of the vacuum gauge field. Thus the *necessary* condition for symmetry breaking is the one-loop effective potential $V_{\text{eff}}^{\text{sca}}$, which is obtained by

$$V_{\text{eff}}^{\text{sca}} = -\frac{1}{2(4\pi)^{\frac{3}{2}}} \int_0^{\infty} \frac{dt}{t^{\frac{5}{2}}} \sum_{l=0}^{\infty} \sum_{n=-\infty}^{\infty} D_l^{\text{sca}}(3) \exp \left[\left(\frac{\Lambda_l^{\text{sca}}(3)}{a^2} - m^2 \right) t \right] \exp \left[-\frac{(n-v)^2}{b^2} t \right], \quad (6)$$

has the non-trivial minimum at non-zero v . Note that this is not a *sufficient* condition. Non-zero v may be equivalent to the trivial vacuum, which depends on the group representation [1].

We use the formula for the elliptic theta function:

$$\sum_{n=-\infty}^{\infty} \exp \left[-\frac{a^2}{b^2} (n-v)^2 t \right] = \frac{b}{a} \sqrt{\frac{\pi}{t}} \sum_{n=-\infty}^{\infty} \exp \left[-\frac{\pi^2 b^2 n^2}{a^2 t} \right] \cos(2\pi n v). \quad (7)$$

Then one can find the one-loop effective potential as

$$V_{\text{eff}}^{\text{sca}} = -\frac{b}{16\pi a^4} \int_0^\infty \frac{dt}{t^3} e^{-(m^2 a^2 - 1)t} \sqrt{\frac{\pi}{t}} \left[\frac{1}{4t} + \sum_{l=0}^{\infty} \left(\frac{1}{2t} - \frac{\pi^2 l^2}{t^2} \right) e^{-\frac{\pi^2 l^2}{t}} \right] \sum_{n=-\infty}^{\infty} \exp \left[-\frac{\pi^2 b^2 n^2}{a^2 t} \right] \cos(2\pi n v). \quad (8)$$

In addition, we use an integral representation of the modified Bessel function:

$$K_\nu(z) = \frac{1}{2} \left(\frac{z}{2} \right)^\nu \int_0^\infty \exp \left(-t - \frac{z^2}{4t} \right) t^{-\nu-1} dt. \quad (9)$$

Then one can rewrite the effective potential as follows:

$$V_{\text{eff}}^{\text{sca}} = -\frac{b}{16\sqrt{\pi} a^4} \sum_{l=0}^{\infty} \sum_{n=-\infty}^{\infty} \cos(2\pi n v) \left[\frac{1}{2} \left(\frac{(m^2 a^2 - 1)}{\pi^2 (b^2/a^2) n^2} \right)^{\frac{7}{4}} K_{\frac{7}{2}} \left(2\sqrt{(m^2 a^2 - 1)(\pi^2 (b^2/a^2) n^2)} \right) + \left(\frac{(m^2 a^2 - 1)}{\pi^2 l^2 + \pi^2 (b^2/a^2) n^2} \right)^{\frac{7}{4}} K_{\frac{7}{2}} \left(2\sqrt{(m^2 a^2 - 1)(\pi^2 l^2 + \pi^2 (b^2/a^2) n^2)} \right) - 2\pi^2 l^2 \left(\frac{(m^2 a^2 - 1)}{\pi^2 l^2 + \pi^2 (b^2/a^2) n^2} \right)^{\frac{9}{4}} K_{\frac{9}{2}} \left(2\sqrt{(m^2 a^2 - 1)(\pi^2 l^2 + \pi^2 (b^2/a^2) n^2)} \right) \right]. \quad (10)$$

We consider the case of the massless scalar field and drop the divergence which is independent of v . Finally, we show the effective potential for the various values of the radius of S^3 in FIG. 1. The shape of the curve depends only on the ratio b/a . The potential minimum is always located at $v = 0$.

B. Fermion field

Next we perform the calculation of the one-loop quantum effect of the fermion field in $M_3 \times S^3 \times S^1$ space-time. The methods of the calculation are similar to the case of the scalar field, except for the eigenvalues of the laplacian and the degeneracies on S^N which are obtained by

$$\Lambda_l^{\text{fer}}(N) = \left(l + \frac{N}{2} \right)^2, \quad D_l^{\text{fer}}(N) = 2^{\frac{N+1}{2}} \frac{\Gamma(l+N)}{l! \Gamma(N)}, \quad (11)$$

respectively.

As was discussed in the previous subsection, the contribution of discrete eigenvalues of S^1 and the vacuum gauge field on S^1 can be taken by considering (5).

Then one can find the one-loop effective potential, setting the bare mass to zero, as

$$V_{\text{eff}}^{\text{fer}} = \frac{b}{(4\pi)^{\frac{1}{2}}a^4} \int_0^\infty \frac{dt}{t^{\frac{7}{2}}} \left[\frac{1}{4t} - \frac{1}{8} + \sum_{l=0}^\infty (-1)^l \left(-\frac{1}{4} + \frac{1}{2t} - \frac{\pi^2 l^2}{t^2} \right) e^{-\frac{\pi^2 l^2}{t}} \right] \sum_{n=-\infty}^\infty \exp \left[-\frac{\pi^2 b^2 n^2}{a^2 t} \right] \cos(2\pi n v), \quad (12)$$

after using the formula for the elliptic theta function (7). This is normalized as a contribution of a single Dirac fermion.

We also drop the divergence independent of v . Finally, we show the effective potential for the various values of the radius of S^3 in FIG. 2. For sufficiently small a/b , the potential minimum appears at $v = 0$.

III. SUMMARY

In this paper, we have calculated the quantum effects of the scalar and fermion field for the vacuum gauge field in $M_3 \times S^3 \times S^1$ space-time. This background geometry can be obtained from the spontaneous compactification of the seven-dimensional gauged supergravity with a positive-definite potential such as the models constructed from the generalized Kaluza-Klein reduction.

We have shown the one-loop effective potential of the scalar field for various values of the radius of S^3 in FIG. 1. We can find that the symmetry breaking itself cannot appear, for any values of the radius, because the effective potential does not have non-trivial minimum at non-zero v . The reason is that there is always the contribution of the zero-mode of S^3 to the effective potential.

On the contrary, for the one-loop effective potential of the fermion field showed in FIG. 2, one can find that there is the possibility of causing the symmetry breaking in some cases, for example, with the fermion in the adjoint representation. The symmetry breaking patterns through the Hosotani mechanism, therefore, change due to the effects of the massive modes of S^3 . This is because there is no zero-mode in the spectrum of the effective theory for the fermion field without the flux coupling.

Although we have considered the toy model started from the motivation about the gauged supergravity, the quantum effects of the effective theory are important for the dynamical symmetry breaking during the cosmological evolution. Further, the supersymmetry breaking effect, leading to the vacuum energy contribution to the cosmological constant, must be taken into consideration.

In the present paper, we have treated only the fermion without the flux coupling, and then there is not a zero-mode in the spectrum. We will further investigate the fermion with a flux coupling having a zero-mode. We anticipate that the symmetry breaking patterns may not change as in the case with the scalar field.

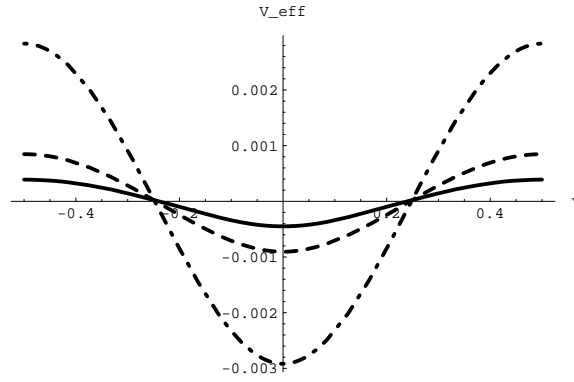


FIG. 1: The one-loop effective potential $V_{\text{eff}}^{\text{sca}}$ of the scalar field is plotted against the effects v of the vacuum gauge field on S^1 . We set the radius of S^1 to unity ($b = 1$). The solid line corresponds to $a = 1$, the dashed line corresponds to $a = 3$, the dot-dashed line corresponds to $a = 5$.

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- [1] Y. Hosotani, Phys. Lett. **B126** (1983) 309; Y. Hosotani, Ann. Phys. (N. Y.) 190 (1989) 233.
 - [2] K. Takenaga, Phys. Lett. **B570** (2003) 244.
 - [3] A. Salam and E. Sezgin, Phys. Lett. **B147** (1984) 47.
 - [4] M. Cvetič, G. W. Gibbons and C. N. Pope, Nucl. Phys. **B677** (2004) 164.
 - [5] J. Kerimo and H. Lü, Phys. Lett. **B576** (2003) 219.

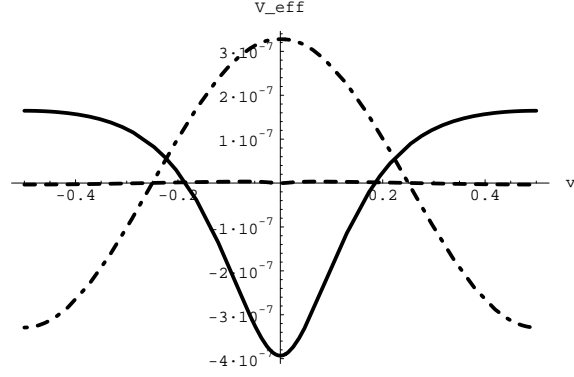


FIG. 2: The one-loop effective potential $V_{\text{eff}}^{\text{fer}}$ of the fermion field is plotted against the effects v of the vacuum gauge field on S^1 . We set the radius of S^1 to unity ($b = 1$). The solid line corresponds to $a = 0.1$, the dashed line corresponds to $a = 0.5$, the dot-dashed line corresponds to $a = 0.7$.

- [6] J. Kerimo, J. T. Liu, H. Lü and C. N. Pope, hep-th/0401001.
- [7] J. Kerimo, J. T. Liu, H. Lü and C. N. Pope, hep-th/0402223.
- [8] I. V. Lavrinenko, H. Lü and C. N. Pope, Class. Quant. Grav. **15** (1998) 2239.
- [9] P. Candelas and S. Weinberg, Nucl. Phys. **B237** (1984) 397.